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First assessment of GPS-based reduced dynamic orbit determination on TOPEX/Poseidon

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Abstract. The reduced dynamic GPS tracking technique has been applied for the first time as part of the GPS experiment on TOPEX/Poseidon. This technique employs local geometric position corrections to reduce orbit errors caused by the mismodeling of satellite forces. Results for a 29-day interval in early 1993 are evaluated through postfit residuals and formal errors, comparison with GPS and laser/DORIS dynamic solutions, comparisons on 6-hr overlaps of adjacent 30-hr data arcs, altimetry closure and crossover analysis. Reduced dynamic orbits yield slightly better crossover agreement than other techniques and appear to be accurate in altitude to about 3 cm RMS.

Introduction

The GPS experiment on TOPEX/Poseidon [*Melbourne et al.*, 1994] presents the first opportunity to apply the reduced dynamic technique for precise orbit determination of earth satellites. The technique exploits the observing strength of GPS to make local geometric corrections to the satellite orbit obtained in a conventional dynamic solution. This reduces orbit errors arising from the mismodeling of forces acting on the satellite while increasing somewhat the effects of measurement error. The principle is illustrated in Fig. 1. The solid line represents a dynamic orbit solution in which the solution trajectory is described by physical and empirical force models; the dashed line represents the observed orbit embodied in the GPS data. A dynamic orbit solution yields postfit residuals that reflect the difference between the solution and the data. Because the flight receiver tracks up to six GPS satellites at once, at each time step there is sufficient residual information to construct geometrically a difference vector between the dynamic solution and the observed orbit.

To implement the geometric correction we model the force acting on the satellite as the sum of a deterministic and a stochastic component, with the latter modeled as a first order Gauss-Markov process [*Wu et al.*, 1991; *Yunck et al.*, 1990]. The data are processed in a sequential filter which estimates the stochastic force at each time step. Since the Gauss-Markov model has a decaying exponential correlation, the stochastic force model is characterized by two selectable parameters: a correlation time constant T and a steady state variance V. When T is set to zero (white noise model) and V is made large, the stochastic correction at each step is unconstrained and completely determined by current data; the resulting position solution is largely geometric, and is little influenced by the model orbit or the previous correction.

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This approach offers a continuum of possible solution strategies. At one extreme we can suppress the geometric correction (set V=0) to yield the classical dynamic solution. At the other extreme we can choose the purely geometric (or kinematic) solution. In that case the underlying dynamic solution serves simply as a point of departure and the effects of force model errors are eliminated. In between we can give arbitrary relative weight to dynamic and geometric information by adjusting T and V, partially reducing dynamic model error. An "optimal" weighting will tend to balance dynamic, geometric, and measurement errors.

Solution Strategy

The results presented here were obtained with JPL's GIPSY/OASIS II analysis software [*Webb et al.*, 1993]. The software is structured as a Kalman filter which processes undifferenced GPS data collected concurrently from the flight receiver and a set of ground receivers. For this analysis, 13 ground sites were used. Data arc lengths of 30 hrs were chosen, with consecutive arcs sharing a 6-hr overlap to permit direct orbit comparisons. The TOPEX/Poseidon orbit solution is obtained as part of a simultaneous solution for many parameters, including GPS orbits; 8 ground site positions (5 are held fixed for reference); receiver and transmitter clock offsets at all but one site, adjusted independently at each time step (white noise clock model); all carrier phase biases; zenith tropospheric delays at each site, adjusted every 5 min as a random walk; and several empirical forces on TOPEX/Poseidon and the GPS satellites.

The TOPEX/Poseidon dynamic solution must first be iterated to convergence. That solution, which adjusts once- and twice-per-rev and constant empirical accelerations in the in-track and cross-track components, converges in two iterations. Table 1 summarizes the dynamic estimation strategy. The reduced dynamic solutions begin with a converged dynamic solution, hold the empirical parameters fixed, and perform a final iteration (with the onboard phase weight tightened from 2 to 1 cm) to estimate the local force corrections. We use a time constant of 15 min and

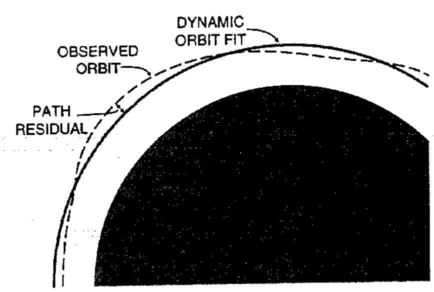


Figure 1. In reduced dynamic tracking, constrained local geometric corrections are applied to an initial dynamic orbit solution.

Table 1. Dynamic Orbit Estimation Strategy

Data intervals:	5-min batches, 30-hr data arcs
Ground data weights:	1 cm phase, 1 m pseudorange
Flight data weights:	2 cm phase, 3 m pseudorange
Adjusted Parameters and A Priori Constraints	
TOPEX/Poseidon state:	1 km, 10 cm/sec, each component
GPS satellite states:	1 km, 1 cm/sec, each component
T/P empirical accelerations:*	1 mm/s^2, each component
GPS solar pressure scale (const):	100%
GPS satellite Y-bias (const):	2 nm/s^2
GPS X, Z scale (stoch):	10% (1 hr update; 4 hr time const)
GPS Satellite Y-bias (stoch):	.1 nm/s^2 (1 hr update; 4 hr t.c.)
Zenith atmospheric delays:	ran. walk (50 cm; 0.17 mm/s$^{1/2}$)
Pole position and pole rate:	5 m; 1 m/day
UT1-UTC rate:	100 sec/day
GPS carrier phase biases:	30,000 km
GPS and receiver clock biases:	1 sec (white noise model)
Eight ground site locations:	1 km each component

*Constant, 1/rev & 2/rev in cross-track & in-track components

steady state sigmas of 10, 20 and 20 nm/s^2 for the radial, cross-track and in-track stochastic forces. Further iteration produces only millimeter changes in the orbit solution.

Results

Here we focus on a 29-day interval beginning on 1 Mar 1993, spanning three 10-day repeat cycles. Reduced dynamic solutions are assessed with several measures, including postfit phase residuals, formal errors, comparison with GPS dynamic solutions, comparison with laser and Doppler dynamic solutions, agreement on orbit overlaps, and altimetry closure and crossover agreement.

Postfit Residuals. The precision of the dual frequency phase observable from the flight receiver is expected to be 3-5 mm, dominated by multipath. If the observing geometry and satellite dynamics were modeled perfectly, the postfit phase residuals at each receiver would be at the level of the data precision. But owing to force model errors, the model trajectory is imperfect and dynamic residuals are higher for flight data. The actual dual frequency RMS residuals for TOPEX/Poseidon, averaged for all 29 days, were 1.2 cm for dynamic and 4.8 mm (or about at the noise level) for reduced dynamic solutions. Ground receiver residuals are about 4.5 mm in both cases. This does not imply that reduced dynamic is a better solution, but residuals at the noise level are expected if the reduced dynamic solution is working properly.

Formal Errors. Formal errors computed by the filter depend on the data weights and a priori errors assigned by the analyst. In the dynamic solution, where model errors dominate, the formal error will be optimistic unless a conservative data weight is used. In the reduced dynamic solution, measurement and model errors are more balanced, and weights reflecting the measurement error yield a truer formal error. A weight of 1 cm was given to both flight and ground phase in the reduced dynamic solutions—double the postfit RMS residuals. The formal RMS errors for a typical solution are 1.5 cm altitude, 2.0 cm cross-track, and 3.4 cm in-track (Fig. 2). In reality, subtle systematic errors, such as mismodeled GPS satellite dynamics, will add to the total error.

Dynamic v Reduced Dynamic. Dynamic and reduced dynamic orbits were computed with the prelaunch JGM-1 gravity model and the JGM-2 model, which was tuned with TOPEX/Poseidon laser and Doppler data at the Goddard Space Flight Center [*Lerch et al.*, 1993]. With JGM-1, the RMS altitude difference

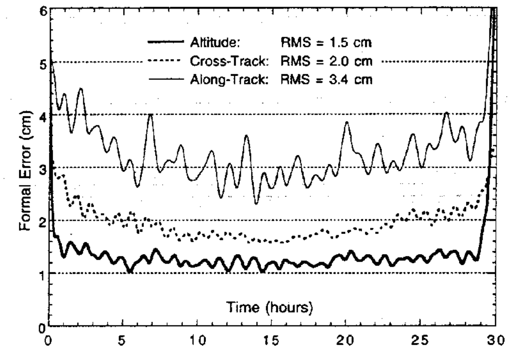


Figure 2. Typical formal errors for reduced dynamic TOPEX/Poseidon orbit solution. Absence of data outside the solution interval causes error growth at edges.

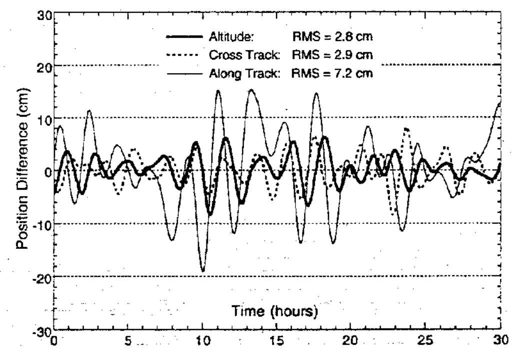


Figure 3. Difference between dynamic and reduced dynamic orbits for a typical solution. Altitude excursions are <10 cm.

between dynamic and reduced dynamic orbits ranges from 3.4 to 4.0 cm. JGM-2 improves this to 2.2-2.9 cm. Figure 3 plots the component differences for a typical JGM-2 solution. The smoothness of the curves and the evident once-per-rev signature suggest that the reduced dynamic solution is primarily correcting dynamic model error rather than introducing measurement error. Figure 4 gives the amplitude spectrum of the dynamic-minus-reduced dynamic altitude over 10 days. The spectrum is typical for dynamic-minus-reduced dynamic solutions and is characteristic of gravity error in a dynamic solution, which because of the Earth's rotation generates tones at 1/rev ± m/day [*Rosborough*, 1986]. The ±2/day tones are larger than expected and may reflect an effect from the 12-hr GPS orbit period. Since we expect little high frequency satellite motion, the reduced dynamic time constant T (15 min) is set to suppress corrections above about 4 cycles/rev. If T is reduced, the high frequency components increase slightly as a result of measurement noise, not real motion. The utility of these comparisons for revealing geographically correlated orbit error is discussed in *Christensen et al.* (1994).

Comparisons with DORIS and lasers. Orbits were also computed with one-way Doppler data from the French DORIS system [*Cazenave et al.*, 1992], which along with laser ranging provides operational precise tracking for the satellite. DORIS is an uplink-only system with about 50 transmitters around the world. Estimated parameters included the satellite state, constant

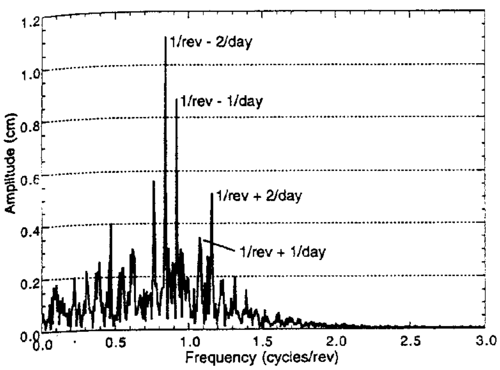


Figure 4. Amplitude spectrum of the altitude difference between dynamic and reduced dynamic orbits over 10 days. Prominent spikes appear at frequencies of 1/rev ± m/day (m≥1), probably as a result of gravity model errors and the daily earth rotation.

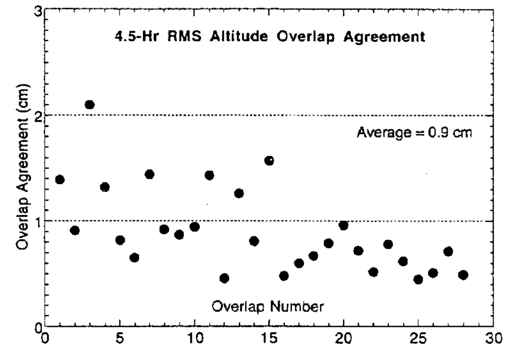
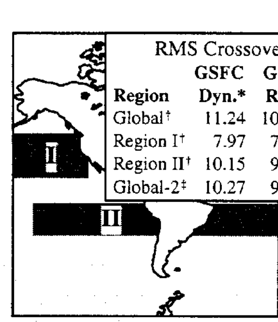


Figure 5. RMS agreement on 4.5-hr orbit overlaps for reduced dynamic solution for all 28 overlaps of the 29-day set.



RMS Crossover Agreement (cm)

Region	GSFC Dyn.*	GPS R. D.	Delta Var.	# of Pts.
Global†	11.24	10.84	8.8 cm²	16833
Region I†	7.97	7.60	5.7 cm²	505
Region II†	10.15	9.14	19.5 cm²	381
Global-2‡	10.27	9.86	8.3 cm²	3205

*laser & DORIS data
† Δt < 10 days
‡ Δt < 24 hrs

Figure 6. Summary of RMS altimetry crossover differences with laser/DORIS dynamic and GPS reduced dynamic orbit solutions. Column 3 gives the difference of the squares of columns 1 and 2.

along-track acceleration, along-track and cross-track once/rev accelerations, zenith tropospheric delays and clock rates at each site for each pass. The typical RMS altitude difference between GPS reduced dynamic and our DORIS dynamic solutions is about 4 cm. Similar results have been obtained by *Schutz et al.* (1994). In another comparison (S. Nerem, personal communication), dynamic solutions produced at the Goddard Space Flight Center with DORIS and laser data showed an RMS altitude agreement of 3.4 cm with the GPS reduced dynamic solution over 20 days.

Overlap Agreement. The daily 30-hr data arcs yield 6-hr solution overlaps. Figure 5 shows the RMS altitude agreement over the central 4.5 hrs of each overlap for the 29 reduced dynamic solutions. The average RMS agreement is 0.9 cm. Since the data are identical on the overlaps and the solution is partly geometric (i.e., local), the principal source of the discrepancy is GPS orbit error. GPS orbits are computed dynamically over each arc and will disagree on the overlaps since their solutions are substantially influenced by non-common data. The GPS discrepancy will then appear in the TOPEX/Poseidon overlap comparison—but scaled down by about 20:1 by the common error cancellation between orbiter and ground receivers. We see in Fig. 5 that the overlap agreement improves over the 30 days. It happened that during the first 10 days up to 7 of the 22 GPS satellites were passing through the earth's shadow on each orbit; by the last 9 days, *no* GPS satellites were eclipsing. The complex dynamics of eclipsing orbits generally degrade their solutions. The average RMS position overlap agreement for the 22 GPS satellite orbits falls from 29 cm for the first 9 days to 19 cm for the last 9 days, which largely explains the improving TOPEX/Poseidon overlaps.

Laser Heights and Altimetry Closure. The Project set up two verification sites equipped with tide gauges and GPS receivers, one on the Harvest oil platform off the California coast and one on Lampedusa island in the Mediterranean. Each site is almost directly overflown by TOPEX/Poseidon every 10 days. To minimize dynamic model error the satellite altitude over the sites was measured with short (10 min) arcs of laser ranging data collected nearby [*Bonnefond et al.*, 1993]. Laser height estimates for 7 Harvest overflights were compared with dynamic and reduced dynamic GPS values. (For several reasons no Lampedusa GPS data were analyzed.) While the dynamic solutions were closer to the lasers in the mean (-7 mm vs 13 mm), both GPS solutions agreed within the expected laser error. The reduced dynamic agreement was more consistent, with a standard deviation of 1.5 cm compared with 2.4 cm for dynamic. In another test, orbit height, tide gauge and GPS ground survey data were combined to give independent estimates of the altimeter range for calibrating the altimeter bias. All orbit solutions—laser, DORIS, and GPS—produced bias estimates of 17-19 cm. While the reduced dynamic estimates again showed the least variation, a larger sample is needed to draw any firm conclusion.

Crossover Analysis. We can test how well ocean height is obtained from the estimated orbit height minus the altimeter height by comparing ocean height data at orbit crossover points. This offers a stringent external test of orbit quality. Since the crossing paths may occur days apart, corrections for known surface variation, such as ocean and solid tides, must be applied. Unmodeled sea height variation caused by atmospheric effects, changes in ocean currents and tide model errors will therefore affect the comparison. As that variation can be large (10 cm or more) compared with orbit error, such a test is in practice less than ideal, but still useful for comparing orbits. From existing maps of sea level variability [e.g., *Koblinsky*, 1988] we identified two ocean regions where the variation over short periods is relatively low and computed crossover agreements for both, as well as globally, for GPS reduced dynamic and GSFC laser/DORIS orbits. The maximum Δt between crossovers was 9.9 days. A second global test was done with a max Δt of 24 hrs. Results are shown in Fig. 6. Reduced dynamic orbits give a consistently lower crossover variance by about 10 cm^2 (or 5 cm^2 for each track, assuming independent ascending and descending tracks). This implies a lower error for the reduced dynamic orbit of 1-2 cm (1σ), depending on the absolute orbit error levels.

Degraded Dynamics. To evaluate reduced dynamic tracking under conditions of poorly modeled dynamics, we have computed the TOPEX/Poseidon orbit with the older GEM-T1 gravity model [*Marsh et al.*, 1988] in place of the tuned JGM-2 model. A dynamic solution with GEM-T1 departs from the JGM-2 solution by 25 cm RMS (77 cm peak) in altitude and 63 cm RMS (1.6 m peak) in position. This is consistent with the expected error based on the GEM-T1 covariance matrix, and is about the level of model error we might face at altitudes of 500-800 km. By contrast, reduced dynamic solutions with the two gravity models (where the GEM-T1 solution is more nearly kinematic) agree to about 7 cm RMS in all components. Purely kinematic solutions with either model depart by 10-12 cm from the optimal JGM-2 solution. To materially improve kinematic performance, more tracking channels and a wider field of view may be needed.

Discussion and Conclusions

Dynamic and reduced dynamic orbit solutions in principle have quite different dominant errors. Random measurement error is small in the dynamic solution, which is limited by the mismodeling of gravity and surface forces. Properly constrained, the reduced dynamic solution will track the motion caused by those forces and be limited instead by measurement and geometrical modeling error. The difference between the two may therefore give a fair upper bound on the true error for each. The evidence at this point does not conclusively favor either approach, although the crossover statistics point to reduced dynamic as the more consistent solution. Formal errors for the reduced dynamic solutions are below 2 cm, but subtle systematic errors may boost the error higher. We believe the evidence supports an estimate of 3 cm RMS for the reduced dynamic altitude error.

The GPS experiment on TOPEX/Poseidon is a work in progress. Enhancements now in development include refined gravity and dynamic tide models, new models for ocean and atmospheric loading, better models of the GPS satellite "noon turn" and dynamics during eclipsing, a new onboard satellite selection algorithm, site- and elevation-dependent data weighting, and an attempt to resolve satellite-to-ground cycle ambiguities. Altitude accuracy may be brought below 2 cm RMS.

Both dynamic and reduced dynamic orbit solutions appear to be accurate to better than 5 cm RMS. The efforts in recent years to refine TOPEX/Poseidon force models have succeeded, and the mission can therefore benefit only modestly from the lower force model sensitivity reduced dynamic tracking offers. But altimetric satellites now being planned for lower altitudes will face a more complex dynamic environment, and dynamic solutions of the present quality will be difficult and costly to attain. Reduced dynamic tracking, with its essential reliance on geometry, should degrade very little and may offer the only practical means of reaching few-centimeter orbit accuracy at low altitudes.

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